

The Derivative of a Power Function

Definition: A power function is a function of the form

$$[f(x)]^n$$

where f is a function and n is a constant.

Examples: $f(x) = x^2$, $h(x) = \sqrt{x}$, $y = (12x + 1)^6$

Find the derivative:

$$f(x) = x^2$$

$$f(x) = x^3$$

$$f(x) = x^4$$

Theorem (The Power Rule, Part I): If n is a natural number, then the function $f(x) = x^n$ is differentiable and

$$\frac{d}{dx} [x^n] = nx^{(n-1)}.$$

For f to be differentiable at $x = 0$, $x^{(n-1)}$ must be defined on an interval containing 0.

Proof of the Power Rule when n is a positive integer greater than 1:

Power Rule when $n = 1$: $\frac{d}{dx} [x] = 1$.

Proof:

Calculate:

$$\frac{d}{dx} (x^3 + x^2 + 1)$$